MI2026

## PROBABILITY AND STATISTICS

Version: 2023.1.0
Objective: The course provides students with the knowledge of probability such as concepts and inference rules of probability as well as random variables and common probability distributions (one-dimensional and two-dimensional); basic concepts of mathematical statistics which help students in dealing with statistical problems in estimation, hypothesis testing, and linear regression. Through the acquired knowledge, students are given a methodology for approaching practical models and finding out an appropriate solution.
Contents: Random events and probability calculation, random variables, probability distributions, random vectors, statistical estimation theory, statistical decision theory, correlation and regression.

## 1. GENERAL INFORMATION

Course name:

Course ID:
Course units:

Expected participants:
Requisites (Prerequisites)

Probability and Statistics
School of Applied Mathematics and Informatics
MI2026

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4(3-2-0-8)
$$

- Lectures: 45 hours
- Tutorial: 30 hours

Third-year students in Undergraduate Advanced Programs

Requisites (Corequisites): - MI1111 or MI1112 or MI1113 (Calculus 1)

- MI1121 or MI1122 (Calculus 2)
- MI1141 or MI1142 (Algebra)


## 2. COURSE DESCRIPTION

The purpose of this course is to acquire basic knowledge of probability theory and statistics as a means to describe and analyze information systems and networks that contain randomness, and to build a foundation that can be used in the graduation thesis, etc. In this course, we will learn the fundamentals of descriptive and inference statistics and acquire the basis of probability theory such as random variables and probability distribution.

## 3. GOALS AND OUTCOMES

At the end of the course, the students should be able to

| Goals/OS | Goals description/OS | Output <br> Standard/Level <br> $(\mathbf{I} / \mathbf{T} / \mathbf{U})$ |
| :---: | :---: | :---: |
| $[\mathbf{1}]$ | [2] | $[\mathbf{3 ]}$ |
| M1 | Understand and be able to do probability problems |  |
| M1.1 | Identify the role of Statistics in the analysis of data from <br> engineering and science. Present graphical and numerical <br> methods for exploring, summarizing and describing data. | ITU |
| M1.2 | Capture principal notions and rules of probability, <br> conditional probability, independent events. Apply the total | ITU |


| Goals/OS | Goals description/OS | Output Standard/Level (I/T/U) |
| :---: | :---: | :---: |
|  | probability formula and Bayes' rule. |  |
| M1.3 | Identify discrete and continuous random variables, their probability distribution (mass probability function and density probability function). | ITU |
| M1.4 | Determine the critical values for well-known distributions: normal distribution, chi-squared distribution, Student tdistribution, and F-distribution | ITU |
| M1.5 | Compute the characteristics: mean, variance, covariance, correlation coefficient. Determine marginal distributions. Recognize the independence. | TU |
| M1.6 | Identify the important role of random samples, their characteristics (sample mean, sample variance,...), particular of a normal sample. Apply the Central Limit Theorem (CLT) and Laws of Large Numbers (LLN). | ITU |
| M1.7 | Estimate parameters and characteristics using point estimators and confidence intervals. | TU |
| M1.8 | Test statistical hypotheses, explain the probability of type I and type II errors. | ITU |
| M2 | Apply probability knowledge to modeling and analysis |  |
| M2.1 | Understand and apply probability to analysis and create some models in real problems | ITU |
| M2.2 | Understand and apply to reading specialized materials | I |
| M2.3 | Recognize simple statistical models and applied them to solve engineering problems | IU |
| M3 | Capacity to synthesize and present a statistics and probability problem as well as understanding responsibility and professional ethics | ITU |
| M3.1 | Capacity to work in groups, write reports and present presentations on the results of homework | TU |
| M3.2 | Understanding responsibilities, professional ethics | IU |

I: Introduce; T: Teach; U: Utilize.

## 4. COURSE MATERIALS

## Textbook

[1] R.E. Walpole, R.H. Myers, S.L. Myers, K. Ye (2011). Probability \& Statistics for Engineers and Scientists. Prentice-Hall (ninth edition).
[2] Applied Mathematics Department (2020). Workbook. Instituted Materials.

## References

[1] Richard, A. Johnson (2005). Probability \& Statistics for Engineers, Person Education, Inc., 2005.
[2] J.S. Milton, J.C. Arnold (2003), Introduction to Probability and Statistics (Principles and Applications for Engineering and the Computing Sciences). McGraw Hill.
[3] J.L. Devore (2000). Probability and Statistics for Engineering and the Sciences. Duxbury.

## 5. GRADING

The overall grade of the course is evaluated throughout the learning process, including three main points: the attendance score (20\%), the midterm test score (30\%), and the final exam score (50\%).

| Assessment <br> Component | Criteria | Assessment <br> Forms | Course <br> Learning <br> Outcomes | Weight |
| :--- | :--- | :--- | :--- | :---: |
| A1. Attendance <br> Score | Student attitude and diligence | Student <br> diligence | M1, M2, M3 | $\mathbf{2 0 \%}$ |
| A2. Midterm <br> Test Score (*) | A2.1. Midterm Test 1 <br> (MTS1, 15 core scale; Content: <br> From week 1 to week 5) | Multiple <br> choice <br> questions | M1.1-M1.3, <br> M2.1 | $\mathbf{3 0 \%}$ |
|  | A2.2. Midterm Test 2 <br> (MTS2, 15 core scale; Content: | M1.3-M1.5, <br> M2.1 <br> From week 6 to week 10) | Writing | M1, M2.1 |

(*) The midterm test score (MTS) is calculated according to the formula MTS $=1 / 3(\mathrm{MTS} 1+\mathrm{MTS} 2)$ and will be adjusted by adding active learning points. Active learning points are worth from -1 to +1 , according to the Higher Education Regulations of Hanoi University of Science and Technology.
6. COURSE TOPICS AND SCHEDULE

| Schedule | Contents | OS | Teaching and learning activities | Assessment |
| :---: | :---: | :---: | :---: | :---: |
| [1] | [2] | [3] | [4] | [5] |
| 1st | Chapter 1. Descriptive Statistics <br> 1.1. Statistics and Engineering <br> 1.2. Population, Sample <br> 1.3. Pictorial and Tabular Methods <br> 1.3.1. Stem-and-Leaf Displays <br> 1.3.2. Histogram <br> 1.4. Measures of Location <br> 1.4.1. The Mean <br> 1.4.2. The Median <br> 1.4.3. Lower Quartile and Upper Quartile <br> 1.4.4. Inter-Quartile Range <br> 1.5. Measure of Variability <br> 1.5.1. Sample Variance <br> 1.5.2. Sample Standard Deviation <br> 1.5.3. Sample Range | $\begin{aligned} & \text { M1.1 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M2.3 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | Teacher: <br> - Giving lectures <br> - Providing <br> lecture notes, assignments - Leading discussions Student in class: <br> - Participating class activities <br> - Answering questions Student at home: | $\begin{gathered} \text { A1 } \\ \text { A2.1 } \\ \text { A3 } \end{gathered}$ |


| Schedule | Contents | OS | Teaching and learning activities | Assessment |
| :---: | :---: | :---: | :---: | :---: |
| [1] | [2] | [3] | [4] | [5] |
|  | Problems for Chapter 1 |  | - Reading documents <br> - Do homework |  |
| $2^{\text {nd }}$ | Chapter 2. Probability <br> 2.1. Sample Space <br> 2.2. Random Events <br> 2.2.1. Complement <br> 2.2.2. Union <br> 2.2.3. Intersection <br> 2.2.4. Mutually Exclusive Events <br> 2.2.5. Mutually Exclusive and <br> Exhaustive Events <br> 2.3. Counting Outcomes <br> 2.3.1. Multiplication Rule <br> 2.3.2. Permutation <br> 2.3.3. Combination <br> 2.3.4. Newton's Formula <br> 2.3.5. Repeated Permutation | M1.2 <br> M2. 1 <br> M2.2 <br> M3.1 <br> M3.2 | Teacher: <br> - Giving lectures <br> - Providing lecture notes, assignments <br> - Leading discussions <br> Student in class: <br> - Participating class activities - Answering questions Student at home: <br> - Reading documents <br> - Do homework | $\begin{gathered} \text { A1 } \\ \text { A2.1 } \\ \text { A3 } \end{gathered}$ |
| $3^{\text {rd }}$ | 2.4. Probability of Events <br> 2.4.1. Definition of Probability <br> 2.4.2. Probability of the Complementary Events <br> 2.4.3. Probability of the Union of Events - Addition Rule <br> 2.4.4. Classical Probability <br> 2.5. Conditional Probability <br> 2.5.1. Definition of Conditional Probability <br> 2.5.2. Multiplication Rules <br> 2.5.3. Probability Tree <br> Problems for Chapter 2 | $\begin{aligned} & \hline \text { M1.2 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ |  | $\begin{gathered} \hline \text { A1 } \\ \text { A2.1 } \\ \text { A3 } \end{gathered}$ |
| $4^{\text {th }}$ | 2.6. Independence <br> 2.6.1. Definition <br> 2.6.2. Properties <br> 2.7. The Total Probability Theorem <br> 2.8. Bayes' Rule <br> Problems for Chapter 2 | $\begin{aligned} & \hline \text { M1.2 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ |  | $\begin{gathered} \hline \text { A1 } \\ \text { A2.1 } \\ \text { A3 } \end{gathered}$ |
| $5^{\text {th }}$ | Chapter 3. Random Variables and Probability Distributions <br> 3.1. Notions of Random Variables <br> 3.1.1. Discrete Random Variables <br> 3.1.2. Continuous Random Variables | $\begin{aligned} & \text { M1.3 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | Teacher: <br> - Giving <br> lectures <br> - Providing <br> lecture notes, | $\begin{gathered} \text { A1 } \\ \text { A2.1 } \\ \text { A3 } \end{gathered}$ |


| Schedule | Contents | OS | Teaching and learning activities | Assessment |
| :---: | :---: | :---: | :---: | :---: |
| [1] | [2] | [3] | [4] | [5] |
|  | 3.2. Cumulative Distribution Function <br> 3.3. Expectation or Mean <br> 3.3.1. Formula for Discrete Random Variables <br> 3.3.2. Formula for Continuous Random Variables <br> 3.3.3. Properties <br> 3.4. Variance <br> 3.4.1. Formula for Discrete Random Variables <br> 3.4.2. Formula for Continuous Random Variables <br> 3.4.3. Properties <br> Problems for Chapter 3 |  | assignments <br> - Leading discussions <br> Student in class: <br> - Participating class activities <br> - Answering questions Student at home: <br> - Reading documents <br> - Do homework |  |
| $6^{\text {th }}$ | Chapter 4. Random Vectors <br> 4.1. Vector of Discrete Random Variables <br> 4.1.1. The Marginal Distributions <br> 4.1.2. How to Calculate Probabilities <br> 4.2. Vector of Continuous Random Variables <br> 4.2.1. The Marginal Distributions <br> 4.2.2. How to Calculate Probabilities <br> 4.3. Independence <br> 4.3.1. Definitions <br> 4.3.2. Properties <br> 4.4. Covariance <br> 4.4.1. Covariance Matrix <br> 4.4.2. Properties <br> 4.5. Correlation Coefficient <br> 4.5.1. Definitions <br> 4.5.2. Properties <br> 4.6. Conditional Distribution <br> 4.6.1. Formula for Discrete Random Variables <br> 4.6.2. Formula for Continuous Random Variables <br> 4.6.3. Properties | $\begin{aligned} & \text { M1.5 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | Teacher: <br> - Giving lectures - Providing lecture notes, assignments <br> - Leading discussions <br> Student in class: <br> - Participating class activities - Answering questions Student at home: <br> - Reading documents <br> - Do homework | $\begin{gathered} \text { A1 } \\ \text { A2. } 2 \\ \text { A3 } \end{gathered}$ |
| $7^{\text {th }}$ | Chapter 5. Several Discrete Distributions <br> 5.1. Uniform Distribution <br> 5.1.1. Definition | $\begin{aligned} & \hline \text { M1.4 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M3.1 } \end{aligned}$ | Teacher: <br> - Giving lectures <br> - Providing | $\begin{gathered} \text { A1 } \\ \text { A2.2 } \\ \text { A3 } \end{gathered}$ |


| Schedule | Contents | OS | Teaching and learning activities | Assessment |
| :---: | :---: | :---: | :---: | :---: |
| [1] | [2] | [3] | [4] | [5] |
|  | 5.1.2. Mean and Variance <br> 5.2. Hyper-Geometric Distribution <br> 5.2.1. Definition <br> 5.2.2. Mean and Variance <br> 5.3. Binomial Distribution <br> 5.3.1. Sequence of Bernoulli Trials <br> 5.3.2. Binomial Distribution B(n, p) <br> 5.3.3. Mean and Variance <br> 5.4. Poisson Distribution <br> 5.4.1. Definition <br> 5.4.2. Mean and Variance <br> 5.4.3. Poisson Process <br> Problems for Chapter 5 | M3.2 | lecture notes, assignments <br> - Leading <br> discussions <br> Student in class: <br> - Participating class activities <br> - Answering questions <br> Student at home: <br> - Reading documents <br> - Do homework |  |
| $8^{\text {th }}$ | ```Chapter 6. Several Continuous Random Variables 6.1. Uniform Distribution on [a, b], \(\mathbf{U}(\mathrm{a}, \mathrm{b})\) 6.1.1. Definition 6.1.2. Mean and Variance 6.1.3. \(\mathbf{U}(0,1)\) 6.2. Exponential Distribution 6.2.1. Definition 6.2.2. Mean and Variance 6.2.3. Poisson Process (continued) 6.3. Gamma Distribution 6.3.1. Definition 6.3.2. Properties 6.3.3. Mean and Variance 6.4. Normal Distribution 6.4.1. Definition 6.4.2. Properties 6.4.3. Mean and Variance 6.4.4. The Standard Normal Distribution 6.4.5. Cumulative Distribution Function of the Standard Normal Distribution 6.4.6. Critical Values 6.4.7. Central Limit Theorem 6.4.8. Normal Approximation for the Binomial Distribution 6.5. Chi-Squared Distribution with (v)``` | $\begin{aligned} & \text { M1.4 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | Teacher: <br> - Giving lectures <br> - Providing lecture notes, assignments <br> - Leading discussions <br> Student in class: <br> - Participating class activities <br> - Answering questions <br> Student at home: <br> - Reading documents <br> - Do homework | $\begin{gathered} \hline \text { A1 } \\ \text { A2.2 } \\ \text { A3 } \end{gathered}$ |


| Schedule | Contents | OS | Teaching and learning activities | Assessment |
| :---: | :---: | :---: | :---: | :---: |
| [1] | [2] | [3] | [4] | [5] |
|  |  |  |  |  |
| $9^{\text {th }}$ | Chapter 7. Random sample <br> 7.1. Notions <br> 7.1.1. Random Sample <br> 7.1.2. Statistic <br> 7.2. Sample Mean <br> 7.2.2. Definition <br> 7.2.3. Mean and Variance <br> 7.2.4. Central Limit Theorem <br> 7.2.5. Laws of Large Numbers <br> 7.3. Sample Variance <br> 7.3.1. Definition <br> 7.3.2. Mean <br> 7.3.3. Convergence Properties <br> 7.4. Sample from Normal Distribution <br> 7.4.1. Definition <br> 7.4.2. Mean and Variance <br> 7.4.3. Properties <br> Problems for Chapter 7 | $\begin{aligned} & \text { M1.5 } \\ & \text { M1.6 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | Teacher: <br> - Giving lectures <br> - Providing lecture notes, assignments <br> - Leading discussions <br> Student in class: <br> - Participating class activities - Answering questions Student at home: <br> - Reading documents <br> - Do homework | $\begin{gathered} \hline \text { A1 } \\ \text { A2.2 } \\ \text { A3 } \end{gathered}$ |
| $10^{\text {th }}$ | Chapter 8. Estimating - Point Estimation <br> 8.1. Problems <br> 8.1.1. Point Estimators <br> 8.1.2. Unbiased Estimator <br> 8.1.3. Consistent Estimator <br> 8.2. Estimating the Population Mean <br> 8.3. Estimating the Population Variance <br> 8.4. Estimating a Proportion or <br> Probability <br> Problems for Chapter 8 | $\begin{aligned} & \text { M1.7 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M2.3 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | Teacher: <br> - Giving lectures - Providing lecture notes, assignments <br> - Leading discussions Student in class: <br> - Participating class activities | $\begin{gathered} \hline \text { A1 } \\ \text { A2.2 } \\ \text { A3 } \end{gathered}$ |


| Schedule | Contents | OS | Teaching and learning activities | Assessment |
| :---: | :---: | :---: | :---: | :---: |
| [1] | [2] | [3] | [4] | [5] |
|  |  |  | - Answering questions Student at home: <br> - Reading documents <br> - Do homework |  |
| $11^{\text {th }}$ | Chapter 9. Confidence Intervals <br> 9.1. Introduction <br> 9.2. Confidence Intervals for the Mean of the Normal Distribution <br> 9.3. Confidence Intervals for the Mean of any Distribution, Large Sample <br> 9.4. Confidence Intervals for a Proportion or Probability <br> 9.5. Confidence Intervals for the Variance of the Normal Distribution <br> Problems for Chapter 9 | $\begin{aligned} & \text { M1.7 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M2.3 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | Teacher: <br> - Giving lectures <br> - Providing lecture notes, assignments <br> - Leading discussions <br> Student in class: <br> - Participating class activities - Answering questions Student at home: <br> - Reading documents <br> - Do homework | $\begin{aligned} & \text { A1 } \\ & \text { A2 } \\ & \text { A3 } \end{aligned}$ |
| $12^{\text {th }}$ | Chapter 10. Testing Statistical Hypotheses <br> 10.1. Introduction <br> 10.1.1. Hypotheses and Tests <br> 10.1.2. Errors of Types 1 and 2 <br> 10.1.3. Critical Region <br> 10.2. Test about the Normal Mean when the Variance is Known <br> 10.3. Test about the Normal Mean when the Variance is Unknown <br> 10.4. Test about the Mean of any <br> Distribution, Large Sample <br> Problems for Chapter 10 | $\begin{aligned} & \text { M1.8 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M2.3 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | Teacher: <br> - Giving lectures <br> - Providing lecture notes, assignments - Leading discussions Student in class: <br> - Participating class activities - Answering | $\begin{aligned} & \text { A1 } \\ & \text { A2 } \\ & \text { A3 } \end{aligned}$ |
| $13^{\text {th }}$ | 10.5. Test about a Proportion or Probability <br> 10.6. Test about the Variance of the Normal Distribution | $\begin{aligned} & \text { M1.8 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M2.3 } \end{aligned}$ | Student at home: <br> - Reading | $\begin{aligned} & \text { A1 } \\ & \text { A2 } \\ & \text { A3 } \end{aligned}$ |


| Schedule | Contents | OS | Teaching and learning activities | Assessment |
| :---: | :---: | :---: | :---: | :---: |
| [1] | [2] | [3] | [4] | [5] |
|  | 10.7. Test about the Difference between two Normal Means <br> 10.8. Test about the Difference between two Proportions <br> 10.9. Test about the Difference between two Normal Variances <br> Problems for Chapter 10 | $\begin{aligned} & \hline \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | documents <br> - Do homework |  |
| $14^{\text {th }}$ | Chapter 11. Regression Analysis <br> 11.1. Simple Linear Regression Model <br> 11.2. Estimating the Model Parameters <br> 11.2.1. Principle of Least Squares <br> 11.2.2. Errors <br> 11.2.3. Estimating Variance <br> 11.2.4. Coefficient of Determination <br> 11.2.5. Confidence Intervals for <br> Parameters <br> 11.2.6. Analysis of Variance <br> Problems for Chapter 11 | $\begin{aligned} & \text { M1.7 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M2.3 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | Teacher: <br> - Giving lectures <br> - Providing lecture notes, assignments <br> - Leading discussions <br> Student in class: <br> - Participating | $\begin{aligned} & \text { A1 } \\ & \text { A2 } \\ & \text { A3 } \end{aligned}$ |
| $15^{\text {th }}$ | 11.3. Regression with Transformed Variables <br> 11.4. Multiple Linear Regression Analysis Problems for Chapter 11 | $\begin{aligned} & \text { M1.7 } \\ & \text { M2.1 } \\ & \text { M2.2 } \\ & \text { M2.3 } \\ & \text { M3.1 } \\ & \text { M3.2 } \end{aligned}$ | class activities <br> - Answering questions Student at home: <br> - Reading documents <br> - Do homework | $\begin{aligned} & \text { A1 } \\ & \text { A2 } \\ & \text { A3 } \end{aligned}$ |

## 7. OTHER REGULATIONS

8. APPROVAL DATE

School of Applied Mathematics and Informatics

